LAPAREX—An Automatic Parameter Extraction Program for Gain- and Index-Coupled Distributed Feedback Semiconductor Lasers, and Its Application to Observation of Changing Coupling Coefficients with Currents

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SUMMARY A reliable and automatic parameter extraction technique for DFB lasers is developed. The parameter extraction program which is named “LAPAREX” is able to determine many device parameters from a measured sub-threshold spectrum only, including gain- and index-coupling coefficients, and spatial phases of the grating at front and rear facets. Injection current dependence of coupling coefficients in a gain-coupled DFB laser is observed, for the first time, by making use of it.

key words: distributed feedback semiconductor lasers, DFB lasers, parameter extraction, coupling coefficient, gain coupling

1. Introduction

Determination of device parameters in distributed feedback (DFB) lasers is very important for optimization of laser characteristics as well as for system design. Among several device parameters, the coupling coefficient is the most important but its determination has only been possible in anti-reflection (AR) coated index-coupled DFB lasers. Moreover, this is not an easy task or not very accurate if there are facet reflectivities remaining [1], [2]. There has been no way to measure the gain-coupling coefficient in gain-coupled DFB lasers, which is the largest issue that needs to be solved before they are practically utilized.

Besides the coupling coefficient evaluation, the spatial phase of the grating should be determined since it affects laser performance significantly when facet reflectivity exists. Nevertheless, there have only been few ways to measure it [3].

The purpose of this paper is to provide an easy and nondestructive parameter extraction method that is applicable to both index- and gain-coupled DFB lasers with facet reflection. The method we present here uses numerical fitting of theoretical sub-threshold spectrum into measured one by the least-square algorithm [4]. The program developed here is named “LAPAREX” which is an abbreviation of “Laser PARameter EXtraction.” In Sect. 2, the model of theoretical sub-threshold spectrum is shown. In Sect. 3, the details of numerical fitting, such as how to determine initial parameters, are explained. In Sect. 4, error estimation method of extracted parameters are given. Section 5 describes observation of changing coupling coefficients in a gain-coupled DFB laser with absorptive grating for the first time [5]. Summary and conclusions are given in the final section.

2. Model

2.1 Spectrum Calculation

The model of theoretical sub-threshold spectrum is based on the static coupled-mode equations [6]:

\[
\frac{\partial R^+}{\partial z} - (\alpha - j\delta)R^+ = -j\kappa e^{-j\theta} R^-
\]

\[
\frac{\partial R^-}{\partial z} - (\alpha - j\delta)R^- = -j\kappa e^{j\theta} R^+
\]

\[
E^+(z) = R^+(z)e^{-j\beta_0 z}
\]

\[
E^-(z) = R^-(z)e^{j\beta_0 z}
\]

where \( z \) is the axial coordinate, \( R^+ \) and \( R^- \) the amplitudes of the forward and backward propagating fields, \( E^+ \) and \( E^- \), \( \beta_0 \) the propagation constant at Bragg wavelength, \( 2\alpha \equiv \Gamma g - \alpha_i \) the net gain, \( \delta \) the deviation of the propagation constant from \( \beta_0, \theta \) the spatial phase of the grating at rear facet (z = 0), and \( \kappa \) the coupling coefficient, respectively.

Equations (1) and (2) are manipulated by the transfer matrix method [7], [8], where \( R^+ \) and \( R^- \) at \( z = z + l \) can be calculated by

\[
\begin{pmatrix}
R^+(z + l) \\
R^-(z + l)
\end{pmatrix}
= \begin{pmatrix}
F_{11}(l) & F_{12}(l) \\
F_{21}(l) & F_{22}(l)
\end{pmatrix}
\begin{pmatrix}
R^+(z) \\
R^-(z)
\end{pmatrix}
\]

in which

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fraction, and Let E by Eqs. (3) and (4), the relations among NAKURA and NAKANO: LAPAREX—AN AUTOMATIC PARAMETER EXTRACTION PROGRAM Here, (3) and (4), the relations among and E by Eqs. (3) and (4), the relations among E±(0), E±(z), and E±(L) become

\[
\begin{bmatrix}
E^+(z) \\
E^-(z)
\end{bmatrix}
= \begin{bmatrix}
F_{11}(z)e^{-j\beta z} & F_{12}(z)e^{-j\beta z} \\
F_{21}(z)e^{j\beta z} & F_{22}(z)e^{j\beta z}
\end{bmatrix}
\begin{bmatrix}
E^+(0) \\
E^-(0)
\end{bmatrix}
= \begin{bmatrix}
a_{111}(z) & a_{112}(z) \\
a_{211}(z) & a_{222}(z)
\end{bmatrix}
\begin{bmatrix}
E^+(0) \\
E^-(0)
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
E^+(L) \\
E^-(L)
\end{bmatrix}
= \begin{bmatrix}
F_{11}(L-z)e^{-j\beta z(L-z)} & F_{12}(L-z)e^{-j\beta z(L+z)} \\
F_{21}(L-z)e^{j\beta z(L+z)} & F_{22}(L-z)e^{j\beta z(L-z)}
\end{bmatrix}
\begin{bmatrix}
E^+_r(z) \\
E^-_r(z)
\end{bmatrix}
= \begin{bmatrix}
a_{111}(L-z) & a_{112}(L-z) \\
a_{211}(L-z) & a_{222}(L-z)
\end{bmatrix}
\begin{bmatrix}
E^+_r(z) \\
E^-_r(z)
\end{bmatrix}.
\]

Here, L is the cavity length. Then, equivalent reflectivity ρ(z), ρr(z) and equivalent transmittance tr(z) at the coordinate z, which are schematically depicted in Fig. 1 are given as:

\[
\rho(z) = \frac{E^+(z)}{E^-(z)} = \frac{r_1a_{111}(z) + a_{112}(z)}{r_1a_{211}(z) + a_{222}(z)}
\]

\[
\rho_r(z) = \frac{E^-_r(z)}{E^+_r(z)} = \frac{-r_1a_{111}(L-z) - a_{112}(L-z)}{r_1a_{211}(L-z) - a_{222}(L-z)}
\]

\[
t_r(z) = \frac{E^+(L)}{E^+_r(z)} = \frac{t_r + (\rho_r^2)tr_t + (\rho_r^2)^2tr_t^2 + \cdots}{1 - \rho_r^2}
\]

in which r1 and r2 are the amplitude reflection coefficients of rear (left-hand-side) and front (right-hand-side) facets. Note that E±r(z) are variables introduced for the sake of convenience. E±r(z) and E±r(z) mean light amplitudes incident into and reflected out of the cavity’s left-hand-side portion (0 to z) at the “virtual” facet z (see Fig. 1) whereas E±r(z) and E±r(z) mean those incident into and reflected out of the right-hand-side portion (z to L) at z.

If spontaneous emission having unit amplitude occurred at the coordinate z, the electric field observed at the front facet due to that emission toward right-hand side should be expressed as

\[
t_r + (\rho_r^2)tr_t + (\rho_r^2)^2tr_t^2 + \cdots = \frac{t_r}{1 - \rho_r^2}
\]

and the electric field due to the spontaneous emission toward left-hand side should be

\[
\rho_r t_r + (\rho_r^2)\rho_r t_r + (\rho_r^2)^2\rho_r t_r + \cdots = \frac{\rho_r t_r}{1 - \rho_r^2}
\]

Assuming that the spontaneous emission is occurring uniformly and randomly throughout the cavity, relative output intensity is calculated by integrating absolute of Eqs. (17) and (18) over z = 0 to L:

\[
\int_0^L |t_r|^2(1 + |\rho_r|^2)(1 - |r_r|^2)dz.
\]

By repeating this calculation for different wavelengths, theoretical sub-threshold spectrum is obtained. Since the sum of the infinite geometric series is used in Eqs. (17) and (18), |ρr| should be less than unity. Therefore, this spectrum calculation is only applicable to “sub-threshold” condition.

2.2 Parameters

There are thirteen parameters involved in the model of sub-threshold spectrum. Some of them are able to be extracted, and others need to be known from the beginning. The parameters that can be extracted by our extraction method are:

- index coupling (IC) coefficient : κi
- gain coupling (GC) coefficient : κg
• parameters associated with net gain profile: \( g_1, g_2, \) and \( \lambda_p \)
• parameters associated with effective refractive index: \( n_{Bragg} \) and \( \frac{dn}{d\lambda} \)
• facet phases of the grating: \( \theta_l \) and \( \theta_r \)

On the other hand, the following parameters need to be fixed:
• cavity length: \( L \)
• grating period: \( \Lambda \)
• facet intensity reflectivities: \( R_l(=|r_l|^2) \) and \( R_r(=|r_r|^2) \)

Here, the net gain profile is assumed to have a parabolic shape, namely,

\[
g(E) = g_1 - g_2 \left( E - \frac{hc}{q_{np}} \right)^2 \tag{20}
\]

and the effective refractive index is assumed to have a linear wavelength dispersion, namely,

\[
n_{\text{eff}}(\lambda) = n_{Bragg} + \frac{dn}{d\lambda}(\lambda - \lambda_{Bragg}). \tag{21}
\]

3. Parameter Extraction Procedure

3.1 Least-Square Algorithm

Numerical fitting is done on the basis of the least-square algorithm. Let \( \mathbf{a} \) indicate parameter vector, like \( \mathbf{a} = (\kappa_i, \kappa_y, g_1, \ldots, y_m(\lambda_i), \ldots, y_c(\lambda_i; \mathbf{a}) \) indicate measured spectrum data at \( \lambda_i \), and \( y_c(\lambda_i; \mathbf{a}) \) indicate calculated spectrum using one parameter set \( \mathbf{a} \). \( N \) is the number of sampling points. Then, the least-square algorithm find the parameter set \( \mathbf{a}_{fit} \) which minimizes \( \sigma^2 \), that is,

\[
\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} [y_m(\lambda_i) - y_c(\lambda_i; \mathbf{a})]^2. \tag{22}
\]

To perform the algorithm, we used the routine called “NL2SOL”, one of general nonlinear least-square programs [9]. This routine takes care of the minimization of \( \sigma^2 \). In doing that, it generally requires two subroutines, one for calculating \( \sigma \), and the other for \( \partial \sigma / \partial \mathbf{a}_k \). Therefore we prepared subroutine that calculates the sub-threshold spectrum \( y_c(\lambda_i; \mathbf{a}) \), and another that calculates its deviation \( \partial y_c(\lambda_i; \mathbf{a}) / \partial \mathbf{a}_k \).

3.2 Initial Parameters Determination

When doing numerical fitting, initial parameters, with which the above mentioned least-square algorithm is started, are very important. Those values need to be close enough to the final value, in order for the fitting values to converge. Success or failure in parameter extraction really depends on the determination of initial parameters. Our initial values are derived by the following procedure:

1. Read peaks and valleys from the measured spectrum, and estimate average Fabry-Perot (FP) mode spacing \( \Delta \lambda_{FP} \) (see Fig. 2).
2. Read stop-band from the widest mode spacing, and regard the center of the stop-band as the Bragg wavelength \( \lambda_{Bragg} \).
3. If cavity length \( L \) and grating pitch \( \Lambda \) are given, the effective refractive index at the Bragg wavelength \( n_{Bragg} \) and its wavelength dispersion \( \frac{dn}{d\lambda} \) are calculated as:

\[
n_{Bragg} = \frac{\lambda_{Bragg}}{2\Lambda} \tag{23}
\]

\[
\frac{dn}{d\lambda} = \frac{n_{Bragg}}{\lambda_{Bragg}} - \frac{\lambda_{Bragg}}{2L\Delta \lambda_{FP}} \tag{24}
\]

The effective refractive index, \( n_{\text{eff}} \), is \( n_{Bragg} + \frac{dn}{d\lambda}(\lambda - \lambda_{Bragg}) \).
4. When both facets are as cleaved, facet relectivities, \( R_{\text{front}} \) and \( R_{\text{rear}} \), are calculated by the following equation and fixed throughout the fitting procedure.

\[
R_{\text{front}} = R_{\text{rear}} = \left( \frac{n_{Bragg} - 1}{n_{Bragg} + 1} \right)^2 \tag{25}
\]

5. From the shortest wavelength among observed FP peaks, which is influenced by DFB mode least, sum of the both facet phases is calculated as

\[
\theta_{\text{rear}} + \theta_{\text{front}} = \mod(\beta \cdot 2L, 2\pi) \lambda_{\text{short}} \tag{26}
\]

where \( \theta_{\text{front}} \) is assumed to be zero as a starting value.
6. From relation between stop-band width and index coupling coefficient with no facet reflectivity calculated beforehand as shown in Fig. 3, the index coupling coefficient can be expressed as

\[
\kappa_i = 0.680255d - 2.6206/L. \tag{27}
\]

The starting value for the index coupling coefficient is calculated using this formula.
Fig. 3  Relation between stop-band $\delta L$ and index coupling coefficient $\kappa_i L$, when there is no facet reflection. The relation is approximated by a linear function shown as an inset.

7. From the peak and valley powers, $P^+$ and $P^-$, of the FP mode with the shortest wavelength, net gain can be calculated as [13]

$$g_1 = \frac{1}{L} \left\{ \ln \frac{\sqrt{10} P^+ /10 - \sqrt{10} P^- /10}{\sqrt{10} P^+ /10 + \sqrt{10} P^- /10} + \ln \frac{1}{\sqrt{R_{\text{front}}}} \frac{1}{\sqrt{R_{\text{rear}}}} \right\}$$

(28)

where the powers are in dB, and $g_2$ in Eq. (20) is zero as an initial value.

8. Gain peak wavelength $\lambda_p$ is set to the Bragg wavelength at the beginning.

9. The initial value of the gain coupling coefficient, $\kappa_g$, is set to zero.

3.3 Results of Parameter Extraction

One good way of checking the reliability of the parameter fitting is to extract parameters from the front facet spectrum and from the rear facet one independently, and compare the corresponding values. Spectra from front and rear facets have different shapes because of asymmetry in the facet phase.

The sample measured here was a 1.55 $\mu$m InGaAsP/InP compressively-strained MQW gain-coupled DFB laser of absorptive grating type, with cleaved facets and a 550 $\mu$m long cavity. Figures 4(a) and (b) show measured spectra as well as calculated ones (initial and fitted) from the front and rear facets of the same device. The initial and final (extracted) parameters are listed in Table 1. Although the fittings were done independently and the shapes of the spectra done independently and the shapes of the spectra were different, the extracted parameters for the front and rear facet spectra agreed well. Reliability of this method is confirmed thereby.

In doing the calculation, all the parameters were assumed to be uniform along the cavity. However, if spatial hole burning occured, this assumption would not hold. In order to check it, field intensity profile along the cavity is calculated using extracted parameters listed in Table 1. In Fig. 5, the field intensity profiles of the two modes on both sides of the stop band are plotted, where one on the shorter wavelength side is indicated by thin solid lines, and the other on the longer wavelength side by dotted lines. Lines with right and left arrows indicate the intensity of forward and

![Fig. 4 Spectra from front (a) and rear (b) facets in a 1.55 $\mu$m InGaAsP/InP GC DFB laser of absorptive grating type (I=24 mA).](image)

The initial and final (extracted) parameters are listed in Table 1. Although the fittings were done independently and the shapes of the spectra were different, the extracted parameters for the front and rear facet spectra agreed well. Reliability of this method is confirmed thereby.

![Fig. 5 Field intensity profiles along the cavity in a GC-DFB laser.](image)

Table 1  Starting and fitted values of parameters in a GC-DFB laser (I=24 mA).

<table>
<thead>
<tr>
<th>Parameter [unit]</th>
<th>front</th>
<th>rear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1$ [cm$^{-1}$]</td>
<td>init.</td>
<td>fitted</td>
</tr>
<tr>
<td>$g_2$ [\mu m$^{-1}$eV$^2$]</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>$\lambda_p$ [nm]</td>
<td>1551</td>
<td>1557</td>
</tr>
<tr>
<td>$n_{B=0}$</td>
<td>3.27</td>
<td>3.27</td>
</tr>
<tr>
<td>$dn/d\lambda$ [\mu m$^{-1}$]</td>
<td>-0.249</td>
<td>-0.250</td>
</tr>
<tr>
<td>$\kappa_i$ [cm$^{-1}$]</td>
<td>10.5</td>
<td>27.9</td>
</tr>
<tr>
<td>$\kappa_g$ [cm$^{-1}$]</td>
<td>0</td>
<td>-4.6</td>
</tr>
<tr>
<td>$\theta_{\text{near}}$ [degree]</td>
<td>106</td>
<td>-177</td>
</tr>
<tr>
<td>$\theta_{\text{front}}$ [degree]</td>
<td>0</td>
<td>149</td>
</tr>
</tbody>
</table>

$$g = g_1 - g_2 \left( E - \frac{\hbar c}{q\lambda_p} \right)^2, \quad n = n_{B=0} + \frac{dn}{d\lambda} (\lambda - \lambda_{B=0})$$
Now that many parameters are derived only from the sub-threshold spectrum, we then estimate how accurate these device parameters are.

First, we define $\sigma_{\min}$ and $\chi$ as:

$$\sigma_{\min}^2 = \frac{1}{N} \sum_i \left( y_m(\lambda_i) - y_c(\lambda_i; a_{fit(0)}) \right)^2$$

(29)

$$\chi^2 = \sum_i \left( \frac{y_m(\lambda_i) - y_c(\lambda_i; a)}{\sigma_{\min}} \right)^2.$$  

(30)

By numerical fitting to the measured data $D_{(0)}$, $a_{fit(0)}$ is determined by minimizing $\chi^2$ through $a$ adjustment. Here, $D_{(0)}$ denotes measurement and fitting without noise disturbance. Then $\chi_{\min}^2$ is obtained as

$$\chi_{\min}^2 = \sum_i \left( \frac{y_m(\lambda_i) - y_c(\lambda_i; a_{fit(0)})}{\sigma_{\min}} \right)^2.$$  

(31)

If random noise is added while measurement, the data set $D$ becomes different and the extracted parameter set $a_{fit}$ is also changed. That is, $a_{fit}$ is distributed around $a_{fit(0)}$. When the values of the parameter set are changed to $a_{fit(0)} + \Delta a$, the value of $\chi^2$ is also changed to $\chi_{\min}^2 + \Delta \chi^2$.

According to the statistics, when noise of these measured data has normal distribution, 99.73% (3$\sigma$ of normal distribution) of $a_{fit}$ is contained within the region of $\Delta \chi^2 = 9$. In our case, the measured spectra do not necessarily have normal distribution. Nevertheless, this model is used in order to assess the error of fitting quantitatively. For example, error ranges of two parameters, $a_1$ and $a_2$, in Fig. 6 are $p \leq a_1 \leq p^*$ and $q \leq a_2 \leq q^*$. Error bars in the following figures are thus calculated.

5. Current Dependence of Coupling Coefficients

Next, we investigated injection current dependence of these device parameters. Figures 7 (a), (b), 8 (a), and (b) are the results corresponding to gain- and index-coupling coefficients, $\kappa_g$ and $\kappa_i$, and front- and rear-facet phases, $\theta_{\text{front}}$ and $\theta_{\text{rear}}$, respectively. Sub-threshold spectra were measured from both front and rear facets, and parameter extraction was carried out on the spectra independently. Resulting fitted parameters for the front and rear facet spectra are shown in the same graphs. Minus sign of $\kappa_g$ indicates “anti-phase” complex coupling.

In Figs. 8(a) and (b), one can see that the facet phases of the grating extracted from front and rear facet spectra agree very well and that they don’t change with current. These results are quite reasonable and let us confirm the reliability of our parameter extraction. In Figs. 9(a) and (b), we notice that the magnitudes of $\kappa_g$ and $\kappa_i$ become small as injection current increases. The change of $\kappa_g$ is considered to be due to saturated absorption of the grating. As the number of photons is increased, absorption coefficient of the grating becomes saturated, thus making $|\kappa_g|$ small. On the other hand, the change of $\kappa_i$ is because of the band-filling effect: as the number of photons absorbed in the grating increases, refractive index of the grating becomes small due to carrier generation. Since the refractive index of...
the absorptive grating is higher originally, this leads to \( \kappa_i \) reduction.

The same measurement was done on an index-coupled DFB laser. The sample measured here was a 1.55\( \mu \)m InGaAsP/InP compressively-strained MQW index-coupled (IC) DFB laser without absorptive grating, with cleaved facets and a 440\( \mu \)m long cavity.

Like in Fig. 4, the sub-threshold spectra for front and rear facets look different in a single device due to asymmetry in facet grating phases in Figs. 9(a) and (b). Nevertheless, the extracted parameters in Figs. 10 and 11 are almost the same for both cases. Moreover, the facet phase values extracted for different currents in Figs. 11(a) and (b) do not differ very much with each other. This again shows the reliability of our program.

It should be noted that the extracted index coupling coefficient, \( \kappa_i \), in Fig. 10(a) does not depend on injection current level unlike before. This is reasonable since the cause of changing \( \kappa_i \) in the previous GC-DFB laser’s case was photon absorption in the absorptive grating, which is not present in this IC-DFB laser. In addition, the extracted value of \( \kappa_g \) in Fig. 10(b) is almost zero, which is another evidence that our parameter extraction method is able to differentiate IC and GC, and to give correct \( \kappa_g \) values.

6. Conclusions

We have developed a reliable, automatic, and nondestructive parameter extraction program for both gain- and index-coupled DFB lasers with finite facet reflectivities, and named it as “LAPAREX” (LAser Parameter EXtraction). This program allows extraction and determination of such parameters as gain- and index-coupling coefficients, and spatial phases of the grating at the front and rear facets, from measured sub-threshold spectra. By making use of this program, injection current dependence of coupling coefficients in a gain-coupled DFB laser of absorptive grating type was detected and measured for the first time. Through the measurement of gain coupling coefficients, which has not been possible so far, structural optimization of gain-coupled DFB lasers would become feasible. Therefore, this program is a key tool for achieving better performance in DFB lasers. The program (for PCs and Macintoshes) can be downloaded and tested from http://www.ee.t.u-tokyo.ac.jp/~nakano/lab/welcome.html.
Fig. 9  Spectra from front (a) and rear (b) facets in a 1.55 μm InGaAsP/InP IC DFB laser (I=3 mA).

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References

Fig. 11 Injection current dependences of front facet phase, $\theta_{\text{front}}$, (a), and rear facet phase, $\theta_{\text{rear}}$, (b) in the 1.55 $\mu$m InGaAsP/InP IC DFB laser.


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